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Steady-State Solution of Catastrophic Feedback Queue Subject to Balking Using Matrix-Geometric Approach

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Abstract: **Queueing theory has grown tremendously over the years with diverse areas of applications, such as business, industry, bank, communication, and hospital etc. The present paper analyses a queueing model where an arriving customer may join the queue with some pre-assumed probability or balk with complementary probability. In addition, if the customer finds the service dissatisfactory, may take feedback. Balking and feedback affect the queue economically; either it causes loss in customer or provides services again. So, studying these parameters is of paramount importance in this competitive market. Further, we have considered inter-arrival time and service time to be exponentially distributed. After constructing the infinitesimal generator matrix, we have obtained the steady-state probabilities using Matrix-Geometric Approach. Various measurable indicators have been evaluated with the assistance of Maple software and based on these measures; we have presented an expected cost and profit analysis.**

Index Terms: **Balking, Catastrophes, Feedback, Markovian-Queues, Matrix-Geometric Approach.**

I. INTRODUCTION

In this fast paced life, it is need of the time for the management dealing with service facility to set up a service mechanism for the optimum use of time during a job; for the maximum growth of that organization or to achieve the set target. Whenever a shared facility needs to be accessed for service by a large number of customers, the simple phenomenon of congestion occurs. Queueing theory provides mathematical solutions to these real life congestion problems. The early literature in queues deals with simple and less complicated queuing models. Since then a lot of new concepts have been used in queueing system such as:- Balking, Reneging, Catastrophes, State dependent service, etc. to show the connectivity to real situations.

Our study is motivated by the applications of balking, feedback and catastrophes in some continuous queueing systems.

Queueing systems incorporating balking have gained a lot of coverage in the sense, how losing a customer implies losing profits. On the other hand queueing systems with feedback represent situations where server has to provide service again, on customer's demand or if the customer finds the service dissatisfactory. They are useful in designing and managing systems like transmission of data, emergency ward of health sector where balking is common and chances of rework is more. The perception of customer impatience was first appeared in the work of Haight (1957). Tackas (1963) analysed a single server queue with feedback. Davingon & Disney (1976) considered single server state dependent feedback queue. Doshi & Jangerman (1986) obtained some important performance measures for an M/G/1 queue where, balking depended on system size using supplementary variable technique. Abou-El-Ata (1991) extended the model of Ancker & Gafarian (1963) to study the state dependent finite queue with impatient customers. For conceivable uses, the history and contributions of researchers on queueing systems with balking and feedback, one may see articles by Santhakumaran & Thangaraj (2000), Choudhury & Paul (2005), Kumar *et al.* (2013), Varalakshmi *et al.* (2018), Bouchentouf e*t al.* (2019).

There has been a surge in the interest in disaster-prone queueing systems. The notion of a disaster striking at any moment, leading to the elimination of customers and the deactivation of the service center can be applied to a variety of real-life issues. Some practical applications of catastrophes include: i) when a server crashes unexpectedly in the manufacturing sector, the waiting units can be forced to abandon the process; ii) a sudden illness/unexpected resignation of a specialised employee in the service sector, etc.. Since, they are very unpredictable in nature; queues with catastrophes have attracted many researchers. In recent years, disaster-prone queueing systems have been investigated by Boucherie and Boxma [1996], Jain and Sigman [1996], Dudin and Nishimura [1999], Artalejo [2000], Kumar and Arivudainambi [2000], Krishna Kumar and Pavai Madheswari [2002], Kumar *et al.* [2007], Thangaraj & Vanitha (2009), Kumar *et al.* (2014), Bura & Bura (2015).

We have used the matrix-geometric technique to obtain steady-state solution to Markovian feedback queue with balking and catastrophes. This method was developed by Neuts (1981). When it comes to solving queueing problems with complex structures, the matrix geometric technique is much more efficient than the traditional probability-generating method. This technique can be used to construct powerful algorithms for a number of practical problems with special structural properties, even if the problems are of higher dimensions.

The primary objectives of this paper are:

1. To obtain steady state solutions to aforesaid queueing system using matrix geometric method.
2. To evaluate important performance measures such as mean number of customers in the system and in the queue, probability of idle, probability of busy, mean balking rate *etc.* and to perform sensitivity analysis.
3. To formulate an expected cost and profit functions based on measures obtained.
4. Graphical representations showing effect of different parameters on expected cost and expected profit functions.

II. Model Assumptions and Descriptions

We consider a Markovian queueing system of infinite capacity, where the arrivals and departures both follow Poisson distribution with mean inter-arrival time $\frac{1}{\lambda}$ and mean inter-service time $\frac{1}{\mu}$. Arriving customer may join the queue with probability '$\boldsymbol{\beta}$'(probability of joining the queue) if one finds the queue non-empty or balk with probability'$\mathbf{1}-\boldsymbol{\beta}$' according to some predetermined norms. If customer, on service completion is satisfied by the service, customer leaves the system with probability '$\boldsymbol{q}$' (disperse probability). On contrary it re-joins the queue with probability '$\boldsymbol{p}$' (feedback probability) if it finds the service dissatisfactory. When disasters strike, all customers are immediately ejected from the system, and the system becomes inactive for a brief period of time. Catastrophes occur according to a Poisson process with a rate of incidence 'ξ' ,when the system is not empty.

The infinitesimal generator matrix Q of the system is given by:

$$\boldsymbol{Q}=\begin{pmatrix} -\lambda & \lambda & 0 & \dots & \dots & \dots \\ (q\mu+\xi) & -(q\mu+\beta\lambda+\xi) & \beta\lambda & 0 & \dots & \dots \\ \xi & q\mu & -(q\mu+\beta\lambda+\xi) & \beta\lambda & 0 & \dots \\ \vdots & \vdots & \vdots & \dots & \dots & \dots \end{pmatrix}$$

Let $n(t) \equiv$ "number of customers in the system at time(t)". Let 'n' be the stationary random variable for the number of the customers in the system. We define π_i= P{n=i}= $\lim_{t\to\infty} P\{n(t)=i\}$, where $i \in \mathbb{W}$ and π_i represents the stationary probability of i customers in the system. The stationary probability vector is given by,

$$\boldsymbol{\pi}=(\pi_0,\pi_1,\pi_2,\dots\dots\dots\dots\dots,\pi_r,\pi_{r+1},\dots\dots\dots) \tag{1}$$

The steady-state probabilities π_i are related geometrically to each other as $\pi_i=\pi_1 R^{i-1}\ \forall i\geq 2$. Here, R is called the rate element and for this system, it is given by:

$$R=\frac{(\beta\lambda+q\mu+\xi)-\sqrt{(\beta\lambda+q\mu+\xi)^2-4\beta\lambda q\mu}}{2q\mu} \tag{2}$$

The steady-state probabilities are obtained by solving the following equations

$$\boldsymbol{\pi Q}=\mathbf{0} \tag{3}$$

$$\boldsymbol{\pi e}=\mathbf{1} \tag{4}$$

III. Performance Measures

We calculate some performance indicators using the probabilities; obtained by employing eq^n (3) & eq^n (4), for the system as follows.

i) "Mean number of customers in the system:"

$$\boldsymbol{MNS}=\sum_{\boldsymbol{n}=\mathbf{1}}^{\infty}\boldsymbol{n\pi_n} \tag{5}$$

ii) "Mean number of customers in the queue:"

$$\boldsymbol{MNQ}=\sum_{\boldsymbol{n}=\mathbf{1}}^{\infty}(\boldsymbol{n}-\mathbf{1})\boldsymbol{\pi_n} \tag{6}$$

iii) Mean Balking Rate (B.R):

$$\boldsymbol{B.R}=(\mathbf{1}-\boldsymbol{\beta})\boldsymbol{\lambda}(\mathbf{1}-\boldsymbol{\pi_0}) \tag{7}$$

iv) Probability that the server is busy:

$$\boldsymbol{P_B}=(\mathbf{1}-\boldsymbol{\pi_0}) \tag{8}$$

v) Probability that the server is idle:

$$\boldsymbol{P_I}=\ \boldsymbol{\pi_0} \tag{9}$$

vi) Mean waiting time in the system:

$$MWS = \frac{MNS}{\lambda} \quad (10)$$

vii) Mean waiting time in the queue:

$$MWQ = \frac{MWS}{\lambda} \quad (11)$$

Special Case

If we put $\beta = 1, q = 1,$ and $\xi = 0$ then the rate element reduces to

$$R = \frac{\lambda}{\mu}$$

and π_n is given by:

$$\pi_n = R^n(1 - R)$$

which is same as the probability of n customers in the system, for classical M/M/1 queue.

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IV. Cost and Profit Model

Constructing an expected cost function for a system which not only get affected by varying arrival and service rates but also by balking, feedback, and catastrophes is very difficult. Let C_1 be the cost associated with a customer present in the queue, C_2 be the cost associated with a customer when server is busy, C_3 be the cost associated with a customer loss (due to balking), and C_4 be the cost associated with server when it is idle. So, we have the expected cost function as,

$$\text{Total Expected Cost } (\mathbf{TEC}) = C_1 * \mathbf{MNQ} + C_2 * P_B + C_3 * \mathbf{B.R} + C_4 * P_I \quad (12)$$

Similarly, for an expected profit function, we have

$$\text{Total Expected Profit } (\mathbf{TEP}) = \rho * \mathbf{MNS} - \mathbf{TEC} \quad (13)$$

where ρ is the revenue.

V. Sensitivity Analysis

In order to arrive at a decision, we carry out sensitivity analyses by substituting different values for the parameters. For calculation, let C_1 =10, C_2 =15, C_3 =20, C_4 =25, and ρ =100. The measurable indicators are computed coupled with total expected cost and total expected profit. Different Cost and profit graphs have been plotted by varying the parameters under consideration. These graphs are illustrated and discussed below.

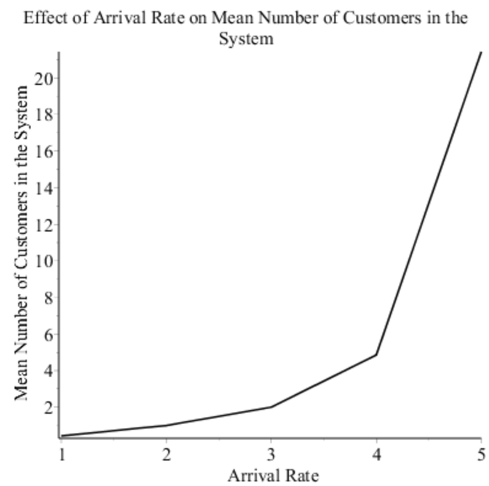

Figure 1. Arrival rate versus MNS

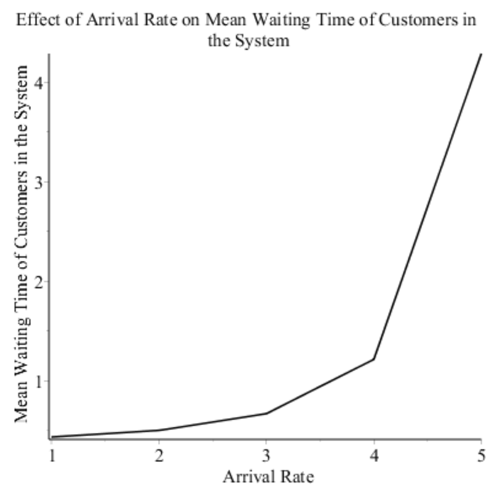

Figure 2. Arrival rate versus MWS

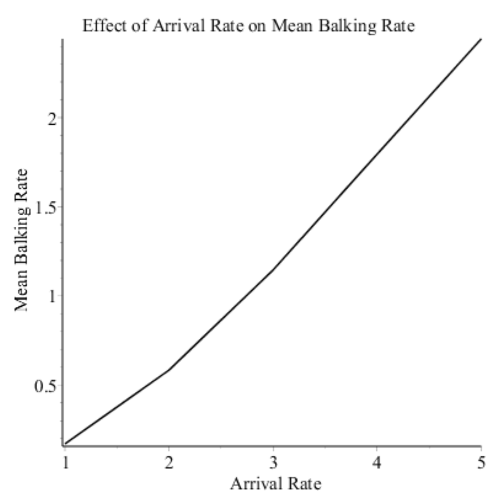

Figure 3. Arrival rate versus B.R

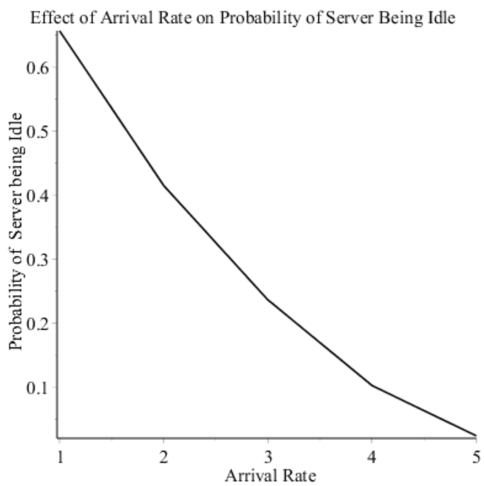

Figure 4. Arrival rate versus Probability of Server being idle

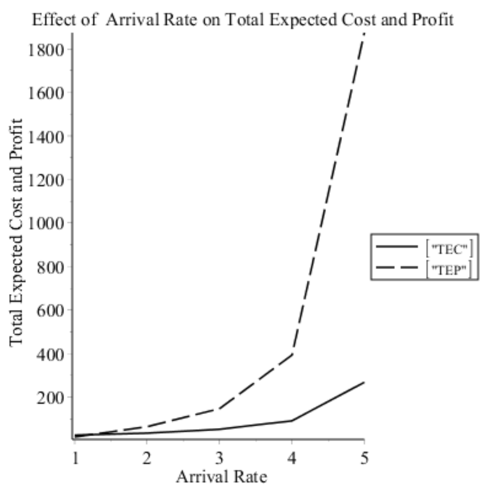

Figure 5. Arrival rate versus TEC and TEP

In figures 1,2,3,4 and 5, we have fixed $\mu = 3$, $\beta = 0.5$, $\xi = 0.01$, and $q = 0.8$ and represented the effect of arrival rate on various performance measures by varying arrival rate. It is clear from the graphs that "mean number of customers in the system", "mean waiting time of customers in the system", "mean balking rate", expected cost and expected profit increase as arrival rate increases. But probability of server, being idle decreases as arrival increases.

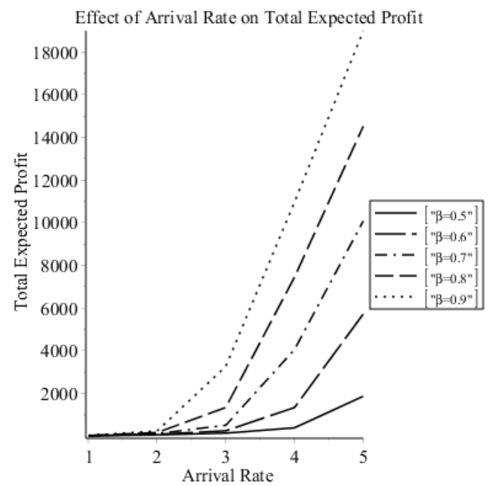

Figure 6. Arrival rate versus TEP for different joining probability.

From figure 6, it is clear from the graph that expected profit increases as arrival rate increases and can be maximized by minimizing probability of balking or by increasing the probability of joining. Less is the balking more is the profit.

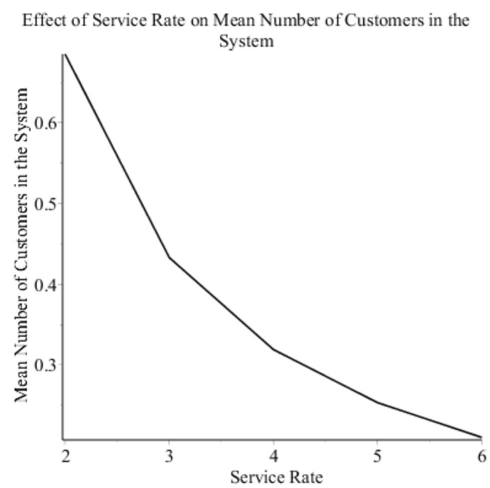

Figure 7. Service rate versus MNS

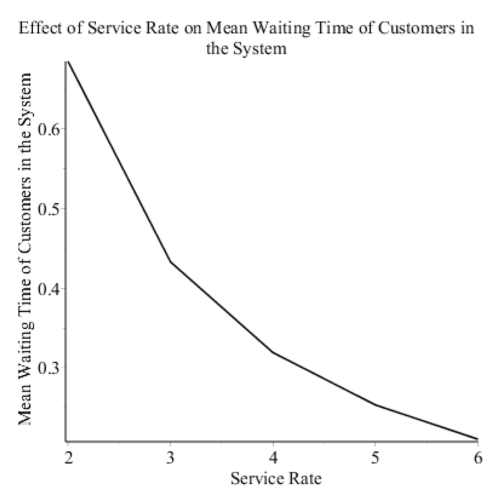

Figure 8. Service rate versus MWS

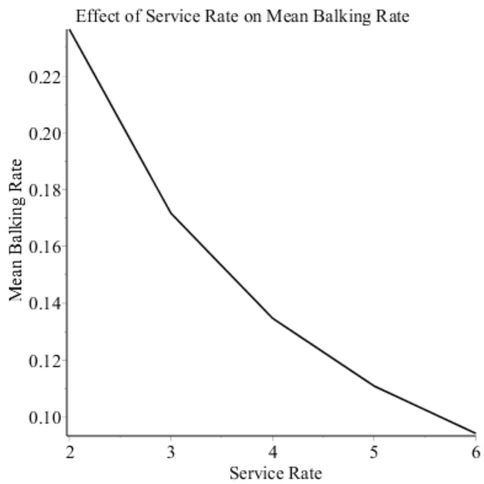

Figure 9. Service rate versus B.R

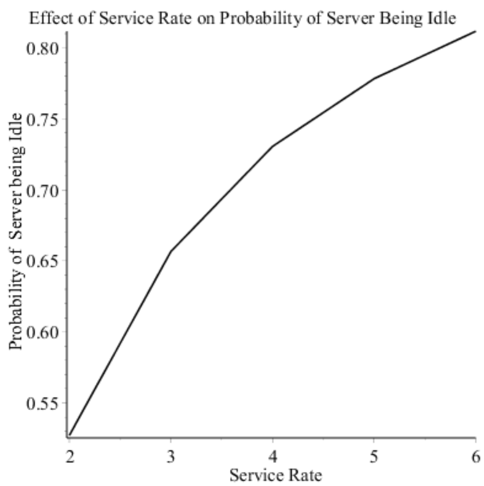

Figure 10. Service rate versus Probability of Server being idle

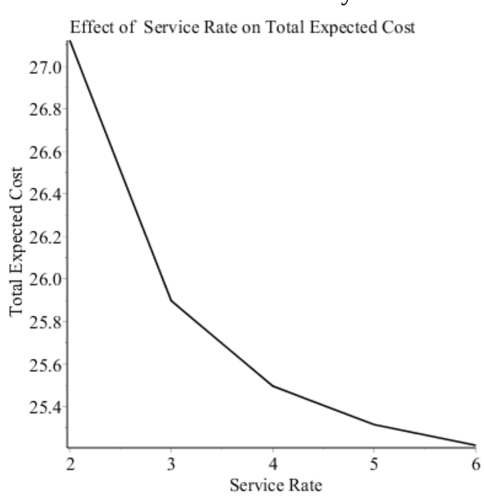

Figure 11. Service rate versus TEC

In figures 7, 8, 9, 10 and 11, we have fixed $\lambda = 1$, $\beta = 0.5$, $\xi = 0.01$, and $q = 0.8$ and represented the effect of service rate on various performance measures. It is clear from the graphs that "mean number of customers in the system", "mean waiting time of customers in the system", mean balking rate, and expected cost decrease as service rate increases. But probability of server, being idle increases as service rate increases.

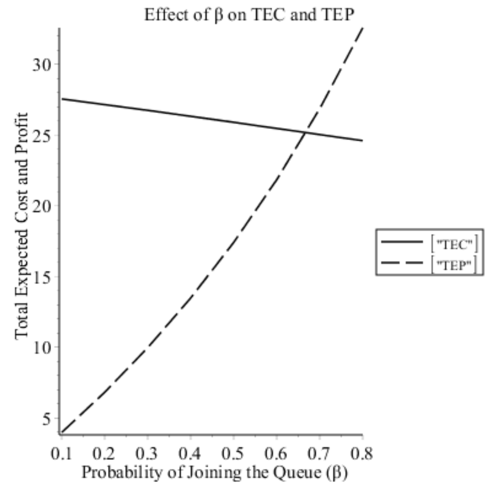

Figure 12. Probability of Joining versus TEC and TEP.

It is clear from the graph that expected cost decreases and expected profit increase as probability of joining increases.

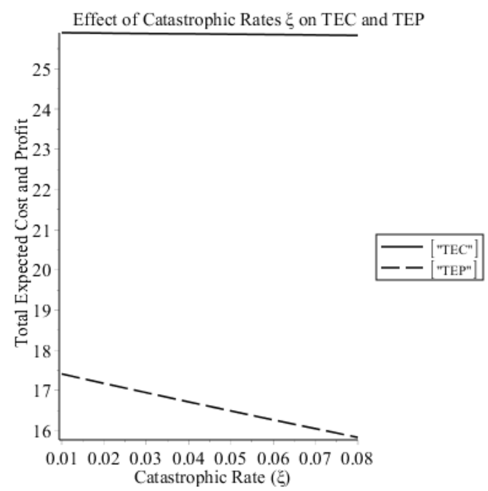

Figure 13. Catastrophic rate versus TEC and TEP.

It is clear from the graph that expected cost and expected profit both decrease as catastrophe rate increases. But it affects profit function more rigorously than the cost function.

CONCLUSION

This paper analyses a single server Markovian feedback queue with balking and catastrophes. The steady-state probabilities of systems have been obtained using matrix geometric approach. We have also developed an expected cost and profit functions and performed sensitivity analyses. We come across many situations where customer's impatience, dissatisfaction or sudden occurrence of any calamity may cause customer loss and affect the system profit as well. Such systems can use the output of the as a tool to improve quality of service and to maximize profit as well. Further, graphs based on numerical analysis, indicate that expected cost can be minimized by providing better quality of service (i.e. minimizing feedback or rework) and by decreasing balking rate.

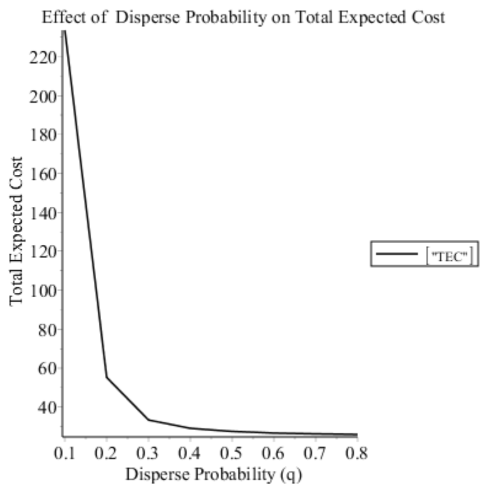

Figure 14. Disperse Probability versus TEC

It is clear from the graph that expected cost decreases if customer decides not take a feedback i.e. expected cost decreases if feedback probability decreases or disperse probability increases.

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